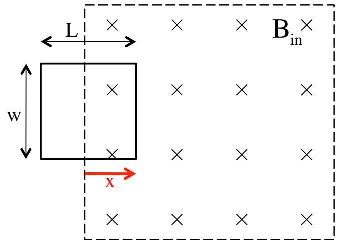
Problem 31.30

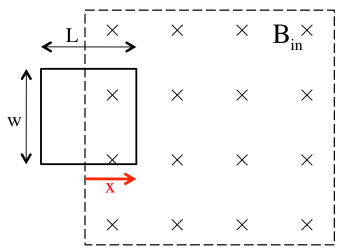


As the coil enters the field, an induced EMF will be generated which will create an induced current in the coil which will interact with the external B-field to generate a force on the coil that FIGHTS WHAT THE COIL IS TRYING TO DO (which, in this case, is to enter the external magnetic field).

AS ALWAYS, start by producing an expression for the magnetic flux through the coil. To do that, assume the coil has entered the field by a distance "x" (see sketch) and, hence, the area of the coil through which flux passes is "wx." With that, we can write:

$$\begin{aligned}\Phi_B &= \vec{B} \bullet \vec{A} \\ &= B\,A\cos 0^\circ \\ &= B\,(wx)\end{aligned}$$

The induced EMF will be:

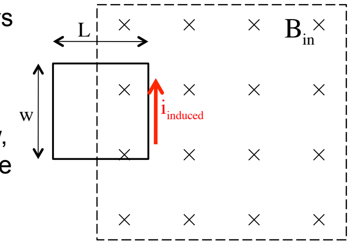


$$\begin{aligned}\varepsilon_{induced} &= -N\frac{d\phi_B}{dt} \\ &= -(1)\,\frac{d(Bwx)}{dt} \\ &= -\,Bw\,\frac{dx}{dt} \\ &= -\,Bw\,v\end{aligned}$$

Using the magnitude of the EMF and the current relationship between the EMF and the resistance of the coil, the force due to the induced current's interaction with the external magnetic field becomes:

$$\begin{aligned}F_{induced} &= i\,\vec{L}_{genericterm} x \vec{B} \\ &= \left(\frac{\varepsilon_{ind}}{R}\right) wB\sin\phi \\ &= \left(\frac{Bwv}{R}\right) wB\sin 90^\circ \\ &= \frac{w^2B^2v}{R}\end{aligned}$$

As for force direction, there are two ways to do this, the hard way and the easy way.

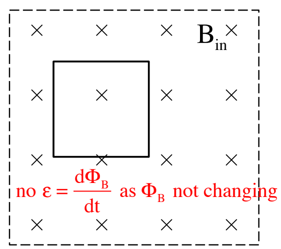


The hard way: According to Lenz's Law, the magnetic flux is INCREASING so the induced B-field will be OPPOSITE the direction of the external B-field, or OUT OF THE PAGE. A coil current that is counterclockwise (using to the cupped-right-hand technique) will provide us with the appropriate B-field.

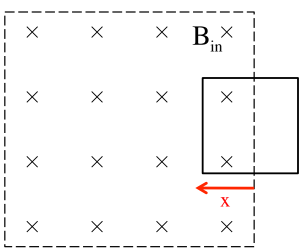
With the direction of current flow, you can use $i\,\vec{L}_{currentSection} x \vec{B}$ to deduce the direction of the force on the wire (that force will be to the LEFT).

The easy way: Induced current-generated induced forces always FIGHT what the coil is doing. In this case, the coil is attempting to enter a magnetic field moving rightward, so the induced force will be LEFTWARD!

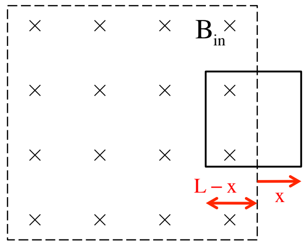
b.) Once the coil is completely inside the magnetic field, there is no longer a CHANGING MAGNETIC FLUX, which means there is no longer an INDUCED EMF or subsequent INDUCED CURRENT, and as such there will be no magnetic force present on the coil.



c.) The only thing that is a little tricky about the coil leaving has to do with the way you define your parameters. If you decide you want to let "x" be associated with the section of coil still in the field (see sketch), then you have to keep in mind that the time derivative of that parameter will be NEGATIVE.



If, on the other hand, you decide to keep the "x" vector in the direction of motion so that it's derivative will be positive, then the amount of coil still inside the B-field becomes "L – x."



As I don't like derivatives that have negative signs hidden within them, I'll use the latter.

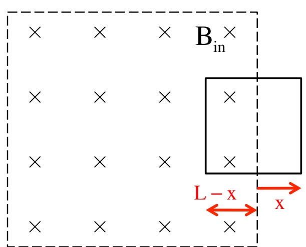
With that: $\Phi_B = B\left(w(L-x)\right)$

and

$$\begin{aligned}\varepsilon_{induced} &= -\frac{d\phi_B}{dt} \\ &= -\frac{d\left(Bw(L-x)\right)}{dt} \\ &= Bw\frac{dx}{dt} \\ &= Bw\,v\end{aligned}$$

and:

$$\begin{aligned}F_{induced} &= i\,\vec{L}_{genericterm} x \vec{B} \\ &= \left(\frac{\varepsilon_{ind}}{R}\right) wB\sin\phi \\ &= \left(\frac{Bwv}{R}\right) wB\sin 90^{o} \\ &= \frac{w^2B^2v}{R}\end{aligned}$$



As the coil is trying to exit the magnetic field moving to the right, the induced force due to the induced current's interaction with the external magnetic field will be LEFTWARD.